Two novel costs for determining the tuning parameters of the Kalman Filter

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***Abstract*—The Kalman filter (KF) and the extended Kalman filter (EKF) are well established techniques for state estimation. However, the choice of the filter tuning parameters still poses a major challenge for the engineers [1]. In the present work, two new costs have been proposed for determining the filter tuning parameters on the basis of the innovation covariance. This provides a cost function based method for the selection of suitable combination(s) of filter tuning parameters in order to ensure the design of a KF or an EKF having an optimally balanced RMSE performance.**

***Index Terms*—Kalman filter, tuning parameters, innovation covariance, cost function**

I. Introduction

The Kalman filter (KF) and its extension for non-linear systems using the linearized system matrices, the extended Kalman filter (EKF), are well established techniques for state estimation which has important applications in various fields like control, monitoring and/or fault detection of various systems and processes. Both the KF and the EKF provide simple yet effective methods for state estimation by accounting for the unmodeled dynamics and measurement noises using the system covariance matrices. The Kalman estimation problem essentially involves the computation of the Kalman filter gain using the estimated system uncertainty and noise covariance matrices. However, it is well-known that a major drawback of these filters is that they require an apriori knowledge about the process and the measurement noise statistics. In most practical applications, the exact information about these statistics is unavailable and so, these covariance matrices, referred to as the filter tuning parameters, have to be supplied by the designer using some ad-hoc procedures [1], [2]. Thus, the choice of the filter tuning parameters still poses a major challenge for the engineers [1], [3].

Several researchers have tried various methods and approaches for choosing the filter tuning parameters to rationalize the ad-hoc nature of the choice. Conventional methods for tuning like downhill simplex numerical optimization algorithm [4] as well as modern techniques like Neural Network (NN), genetic algorithm and fuzzy logic based approaches [5], [6], [7] have been used for the KF tuning problem. Rosendo et al. [8] designed a self-tuned Kalman filter and compared it with the conventional running average and a conventionally tuned Kalman filter. These filters were used in the low-pass filtering stage required in some active power filter algorithms and their results showed that though all the three methods perform well, but the self-tuned Kalman filter reacts faster under transient conditions. A method using the normalization of the system matrices has been used for the choice of the covariance matrices for the online determination of rotor position and speed of a permanent-magnet synchronous motor in [9]. However, none of these methods have been able to provide a deterministic method for selecting the filter covariances. Some researchers have focussed on innovations in order to address the filter tuning issue. One method involves the use of the estimated autocovariance of the output innovations to compute a least-squares estimate of the noise covariance matrices [10]. Kailath [11] also proposed that the innovations be measured and their mean and covariance be approximated using statistical methods in order to verify the KF performance and to adjust the KF parameters to improve the performance of the state estimation. He further stated that if the mean and covariance of the innovations are not as expected then it indicates that

the choice of any or all of the system matrices as well as the covariances is/are incorrect. An innovation based cost function for KF, termed as the normalized innovation squared (NIS), has been suggested in [12] which uses the UMPITS [13] to find out whether the particular choice of the tuning parameters can assure convergence of the filter. This cost is defined in terms of the innovation q_k and the innovation covariance S_k as

$$J = \frac{1}{N} \sum q_k^T (S_k)^{-1} q_k \tag{1}$$

Under the hypothesis that the filter is consistent, the NIS has a chi-square distribution with n_z degrees of freedom, where n_z is the dimension of the measurements. One of the limitations of this method is that it has to be tested online and so it cannot be used for predictions of suitable choices of the filter tuning parameters but can only be used for verification of dimensional consistency of the filter.

In the present paper, a cost function based predictive method, for the selection of suitable combination(s) of filter tuning parameters, has been proposed in order to ensure the design of a Kalman filter having a judiciously balanced performance in terms of robustness and sensitivity. For this purpose, two novel performance indices (costs) have been suggested which can be used to predict and/or compare the quality of the practically obtained RMSE performances. In Section II, the new performance indices have been derived and their significance has been discussed. A realistic problem, used to demonstrate the effectiveness of the proposed costs, is stated in detail in Section III, while the corresponding simulations and results are stated in Section IV. Conclusions are stated in Section V.

II. Novel Performance Indices for the choice of Filter Tuning Parameters

A linear discrete time (stochastic) system may be described using linear(ized) state and observation equations, at a particular time instant k, as follows:

$$\begin{aligned} x_{k+1} &= F_k x_k + G_k u_k + w_k \\ y_k &= H_k x_k + v_k \end{aligned} \tag{2}$$

Here, u_k is the known input, while w_k and v_k are the state and measurement noises. Both of these are zero-mean white noises with their uncorrelated covariances being Q_k and R_k respectively. F_k, G_k and H_k are the state transition, input and measurement matrices respectively.

The Kalman filter, which estimates the state vector x_k from the measurements y_k in an optimal sense, can be expressed as a set of sequential equations for the apriori state estimate and error covariance, $\hat{x}_k^-$ and P_k^-, the innovation and its covariance, q_k and S_k, the Kalman Gain, K_k, and the aposteriori state estimate and error covariance $\hat{x}_k^+$ and P_k^+ as follows:

$$\hat{x}_k^- = F_{k-1}\hat{x}_{k-1}^+ + G_{k-1}\hat{u}_{k-1} \tag{3}$$

$$P_k^- = F_{k-1}P_{k-1}^+ F_{k-1}^T + Q_{k-1} \tag{4}$$

$$q_k = y_k - H_k \hat{x}_k^- \tag{5}$$

$$S_k = H_k P_k^- H_k^T + R_k \tag{6}$$

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \tag{7}$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k q_k \tag{8}$$

$$\begin{aligned} P_k^+ = {} & (I - K_k H_k)[F_{k-1}P_{k-1}^+ F_{k-1}^T + Q_{k-1}] \\ & (I - K_k H_k)^T + K_k R_k K_k^T \end{aligned} \tag{9}$$

For the design of a suitable KF, four tuning parameters need to be determined prior to implementing the filter. These are the initial state estimate x_0 and the three uncertainty or noise covariance matrices, namely the initial state (estimation) error covariance P_0, the process (model) noise covariance Q and the measurement noise covariance R. Of these covariances, P_0 is only the initial choice of the state estimation error covariance P_k, which has to be decided by the designers, since the state estimation error covariance P_k changes as the filtering progresses with time and is expected to reach a steady value as the filter converges, provided that the system is not subjected to any major change in the system input. However, Q and R have to be decided for the total duration of filtering and depending on practical considerations, designers choose these to be time-invariant or time-varying.

The choice of the elements of x_0 and P_0 may range from infinitely large values to small values depending on the available information about the relevant states [14]. It has further been observed that the choices of x_0 and P_0 mainly affect the initial part of the estimation so these are usually not very critical unless the initial estimation exceeds acceptable limits [14]. The choice of R is also non-critical since the sensor characteristics, which are usually known beforehand, can be used to decide on a suitable matrix.

Among all the tuning parameters, the tuning of the process noise covariance Q is considered to be the most critical [1]. This is so since all the model uncertainties and inaccuracies as well as the noises affecting the process are incorporated quantitatively into Q. It is also known that a *proper* ratio of

the filter tuning parameter values affects the filter performance [12], [14]. This further complicates the choice of a suitable Q.

In the present work, the innovation q_k, which directly affects the Kalman gain, has been identified as the critical parameter which can be utilized to predict the proper choice of Q for the filter, given an arbitrary choice of x_0, P_0 and R, which may or may not be identical to the values obtained from the actual system. This use of q_k can be justified from the established fact that the Kalman gain plays a major role in ensuring the optimized performance of the KF while the NIS ensures filter consistency [1], [3], [12]. However, the innovations, which are random variables obtained in real-time, are not quite helpful for predictions due to lack of a deterministic basis.

So, in order to obtain a deterministic basis for the prediction of the *suitable* filter tuning parameters, the innovation error covariance S_k, as stated in eqn (6), is studied instead. It is observed that S_k depends on the aposteriori estimation error covariance *for the measured outputs* (not states), namely $H_k P_k^+ H_k^T$, which can be obtained as follows.

Starting with the expressions for the apriori state estimation error covariance P_k^- and the Kalman gain K_k from eqns (4) and (7) respectively, we obtain

$$H_k K_k = (A+B)(A+B+R_k)^{-1} \tag{10}$$

where $A = H_k F_{k-1} P_{k-1}^+ F_{k-1}^T H_k^T$ and $B = H_k Q_{k-1} H_k^T$.

The aposteriori state estimation error covariance P_k^+, as stated in eqn (9), is pre- and post-multiplied by H_k and H_k^T respectively, and then pre-multiplied by $(A+B)^{-1}$, where A and B are as defined in the previous equation, to obtain

$$\begin{aligned}(A+B)^{-1}&[H_k(P_k^+ - F_{k-1}P_{k-1}^+F_{k-1}^T)H_k^T] \\ &= (A+B)^{-1}B - (A+B+R_k)^{-1}(A+B) \\ &= (A+B)^{-1}B + (A+B+R_k)^{-1}R_k - I_m. \end{aligned} \tag{11}$$

Taking the trace of both sides of eqn (11) and rearranging, we obtain the two new costs J_{1k} and J_{2k} as

$$J_{1k} + J_{2k} \quad = \quad m - trace\{N_k\} \tag{12}$$

where

$$\begin{aligned} J_{1k} &= trace\{(A+B+R_k)^{-1}R_k\} \\ J_{2k} &= trace\{(A+B)^{-1}B\} \\ \text{and } N_k &= (A+B)^{-1}[H_k(F_{k-1}P_{k-1}^+F_{k-1}^T - P_k^+)H_k^T]. \end{aligned}$$

It is useful to note from eqn (12) that the value of $J_{1k} + J_{2k}$ at any instant k deviates from the number of measurements m due to the contribution of the term $trace\{N_k\}$ at that particular instant of time k.

In order to appreciate the significance of the two new costs, it must be noted that the focus of the present work is on the estimated *measurement* and the factors contributing to it while the standard treatment in the existing literature [12], [15], [16] focuses simply on the estimated states and the errors thereof. Here, both the apriori state estimation covariance $F_k P_{k-1}^+ F_k^T$ and the process noise covariance Q_{k-1} have been projected onto the innovation covariance S_k and so, they affect the measurement estimate as A and B respectively, as seen from eqns (4), (6) and (10).

For the evaluation of the overall filter performance, let the performance indices or costs J_1 and J_2 and a controlling parameter for the costs, n_q, be defined in terms of the total horizon N as

$$\begin{aligned} J_1 &= \frac{1}{N}\sum_{k=1}^{N} J_{1k} \\ &= \frac{1}{N}\sum_{k=1}^{N} trace\{(A+B+R_k)^{-1}R_k\} \\ J_2 &= \frac{1}{N}\sum_{k=1}^{N} J_{2k} \\ &= \frac{1}{N}\sum_{k=1}^{N} trace\{(A+B)^{-1}B\} \\ \text{and } n_q &= \frac{1}{N}\sum_{k=1}^{N} log\{trace(B)\}. \end{aligned} \tag{13}$$

In order to evaluate the performances of the KF and the EKF, the four filter tuning parameters x_0, P_0^+, R may be fixed apriori, as stated earlier in this Section, while Q_0 is continuously varied. A change in Q_0 changes the overall n_q, irrespective of the choice of a fixed or variable Q_k. Furthermore, this change in Q_0 affects the general tuning parameter ratios P/Q and Q/R at all instants and for all the states. As stated in [12], these affect the overall filter performances.

The following are observed from eqn (13) for a changing Q_0, and hence a changing n_q.

(i) For very large Q_0, B is much larger than R in terms of the trace, and so, J_1 tends to 0.
Also in this case, J_2 tends to the number of measurements m for a convergent and small A.

(ii) Similarly, when Q_0 is small, B is significantly smaller than R in terms of the trace, then $J_1 \rightarrow m$ and reaches a steady value.

(iii) When B is comparable to R in terms of the trace, then J_1 and J_2 change significantly from both the upper and lower bounds stated earlier.

III. Case Study

In order to verify the predictability of the performance of a filter for different combinations of the tuning parameters using the defined costs J_1 and J_2, the problem considered in the present work is that of the tracking of a 2D ballistic target as discussed in [17]. It is assumed that the object enters the atmosphere in reentry phase under the presence of nonlinear air drag as well as gravity and so, the trajectory of the target is a nonlinear one.

The equivalent discrete-time target motion can be expressed using the nonlinear state equation [17]

$$\begin{aligned} x_{k+1} &= f(x_k) + Gu_k + w_k \\ &= Fx_k + Gf_{kk}(x_k) + Gu_k + w_k \end{aligned} \tag{14}$$

where the state vector $x_k = [x_{1k}\, \dot{x}_{1k}\, x_{2k}\, \dot{x}_{2k}]^T$ consists of the positions in the x and y directions denoted as x_{1k} and x_{2k} respectively and their corresponding velocities. $u_k = [0\ (-g)]^T$ and $w_k = N(0, \sqrt{Q_t})$ are the input matrix and the process noise respectively.

The function $f_{kk}(x_k) = -\frac{1}{2\beta}\rho g\sqrt{\dot{x}_{1k}^2 + \dot{x}_{2k}^2}\ [\dot{x}_{1k}\ \dot{x}_{2k}]^T$, while

$$Q_t = \begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix}.$$

These and all other terms, as used in the present problem, are as defined in [17]. Only, the target ballistic co-efficient β has been assumed to be known with a constant value of $\beta = 40000 kgm^{-1}s^{-2}$. The acceleration due to gravity g is assumed to be constant at $9.81 ms^{-2}$.

The ballistic target trajectory has been generated considering the initial value of the state vector as $x_0 = [232km\ \ 2.29cos(190°)kms^{-1}\ \ 88km\ \ 2.29sin(190°)kms^{-1}]^T$ and the air density function $\rho = C_1 e^{-C_2 x_2}$ with
$C_1 = 1.227,\ C_2 = 1.093 \times 10^{-4}$ when $x_2 < 9.144km$
and $C_1 = 1.754,\ C_2 = 1.490 \times 10^{-4}$ when $x_2 > 9.144km$.
This change in the air density during the reentry becomes very crucial during the estimation as the system nonlinearity changes abruptly at this height.

The measurement equation is considered to be

$$y_{mk} = h(x_k) + v_k \tag{15}$$

where the terms have their usual meaning and the measurement noise is considered to be $v_k = N(0, \sqrt{R_A})$.
In the practical scenario, the measurements available depend on the choice of the sensor(s) and other practical constraints. In order to obtain the measurements of the positions x_{1m} and x_{2m}, the simulated radar measurements of range r and angle ε, available in polar co-ordinates, are converted into Cartesian coordinates. The relations used are $x_{1m} = rcos(\varepsilon)$ and $x_{2m} = rsin(\varepsilon)$. These data are further corrupted with the randomly generated zero-mean measurement noise v_k using the noise covariance matrix R_A given as

$$R_A = \begin{bmatrix} \sigma_d^2 & \sigma_{dh} \\ \sigma_{dh} & \sigma_h^2 \end{bmatrix} \tag{16}$$

where

$$\begin{aligned} \sigma_d^2 &= \sigma_r^2\, cos^2(\varepsilon) + r^2\, \sigma_\varepsilon^2\, sin^2(\varepsilon) \\ \sigma_h^2 &= \sigma_r^2\, sin^2(\varepsilon) + r^2\, \sigma_\varepsilon^2\, cos^2(\varepsilon) \\ \text{and } \sigma_{dh} &= (\sigma_r^2 - r^2\, \sigma_\varepsilon^2)\, sin(\varepsilon)\, cos(\varepsilon) \end{aligned}$$

such that $\sigma_r = 100m$ is the variance of r and $\sigma_\varepsilon = (.017/57.3)^\circ$ is the variance of ε.

IV. Simulation and Results

In order to evaluate the performances of the KF and the EKF, the four filter tuning parameters x_0, P_0^+, R and a nominal Q_0, henceforth referred to as Q_{nom}, have to be fixed. In this case, x_0 and P_0^+ are obtained using the two point differencing method as stated in [12] and [17], R is obtained from the sensor and is considered to be time-invariant while Q_{nom} is obtained as the adaptive Q as discussed in [3]. The values of these tuning parameters, which are considered to be the same for both KF and EKF, are as follows:

$$x_0 = \begin{bmatrix} 2.25 \times 10^5 \\ -2.81 \times 10^3 \\ 9.26 \times 10^4 \\ 6.75 \times 10^3 \end{bmatrix}$$

$$R = \begin{bmatrix} 10.54 & -3.85 \\ -3.85 & 37.15 \end{bmatrix}$$

$$P_0^+ =$$

$$\begin{bmatrix} 2.48\times10^6 & 0 & -6.76\times10^6 & 0 \\ 0 & 1.24\times10^6 & 0 & -1.73\times10^6 \\ -6.76\times10^6 & 0 & 1.47\times10^7 & 0 \\ 0 & -0.73\times10^6 & 0 & 7.34\times10^6 \end{bmatrix}$$

$$Q_{nom} =$$

$$\begin{bmatrix} 2.48\times10^5 & 6.32\times10^4 & -5.10\times10^5 & -1.04\times10^5 \\ 6.32\times10^4 & 2.34\times10^4 & -1.04\times10^5 & -2.88\times10^4 \\ -5.10\times10^5 & -1.04\times10^5 & 1.44\times10^6 & 3.45\times10^5 \\ -1.04\times10^5 & -2.88\times10^4 & 3.45\times10^5 & 1.20\times10^5 \end{bmatrix}$$

In order to predict the performances of the KF and the EKF and compare them for different combinations of the tuning parameters, the costs J_1 and J_2 have been obtained for varying n_q as shown in Fig.1 and Table I by using $Q_0 = 10^p Q_{nom}$, and varying p suitably, as evident from the first two columns of the Table. These predictive costs have been compared with the RMSE performances of the KF, (Fig.2), and the EKF, (Fig.3), obtained using the same tuning parameters.

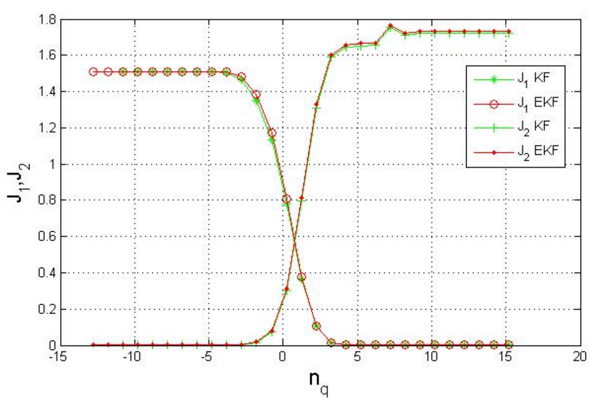

Fig. 1. Plots of J_1 and J_2 vs. n_q for KF and EKF

As can be predicted from the derivations, it is observed from Fig.1 and Table I that, for changing Q_0 and hence, for changing n_q,

1) Both J_1 and J_2 are bounded by the number of measurements $m = 2$ in the upper limit and 0 in the lower limit for significantly high and low values of n_q. As n_q changes, the values of both J_1 and J_2 change between these limits of m and 0.

2) While P_0 and R are fixed, J_1 attains a steady maximum value close to m for low n_q while J_2 also reaches a steady maximum value close to m, but for high n_q. In this case, it is observed that J_1 attains a steady value of 1.51, for KF as well as EKF, for values of n_q less than -4.79 and -3.79 respectively while J_2 attains a steady value of 1.72 and 1.73 for KF and EKF respectively, for n_q larger than 9.21 in both the cases. A minimum value, very close to 0, is obtained for J_2 when n_q is very small, typically -2.79 or less, and for J_1, when n_q is very large, typically 4.21 or more, for both KF and EKF.

3) At a particular value of n_q, with the corresponding value of Q_0 henceforth referred to as the compromise value, Q_{comp}, there is a crossover of the plots for J_1 and J_2. In this case, the corresponding value of n_q for Q_{comp} lies between 0.21 and 1.21 for both KF and EKF.

4) The filter exhibits robustness for those combinations of the tuning parameters for which the value of J_2 is close to the number of measurements m. On the other hand, when the value of J_1 is close to the number of measurements, this indicates a sensitivity in the RMSE performance but this might also cause the filter to diverge if the actual system noise or disturbances

TABLE I
COSTS J_1 AND J_2 FOR KF AND EKF

p	n_q	J_1 KF	J_2 KF	J_1 EKF	J_2 EKF
-13	-6.79	1.51	0.00	1.51	0.00
-12	-5.79	1.51	0.00	1.51	0.00
-11	-4.79	1.51	0.00	1.51	0.00
-10	-3.79	1.50	0.00	1.51	0.00
-9	-2.79	1.46	0.00	1.48	0.00
-8	-1.79	1.35	0.01	1.39	0.02
-7	-0.79	1.13	0.07	1.17	0.08
-6	0.21	0.78	0.30	0.81	0.31
-5	1.21	0.37	0.80	0.38	0.81
-4	2.21	0.10	1.31	0.11	1.33
-3	3.21	0.02	1.59	0.02	1.60
-1	4.21	0.00	1.65	0.00	1.66
0	5.21	0.00	1.65	0.00	1.66
1	6.21	0.00	1.65	0.00	1.67
2	7.21	0.00	1.75	0.00	1.76
3	8.21	0.00	1.71	0.00	1.72
4	9.21	0.00	1.72	0.00	1.73
5	10.21	0.00	1.72	0.00	1.73
6	11.21	0.00	1.72	0.00	1.73

are large. Hence, it can be said that the robustness of the RMSE performances increase for KF and EKF for $n_q \geq 1.21$ while sensitivity of the RMSE performances of the filters increase for $n_q < 1$. This is validated from the filter performances as observed in Figs. 2 and 3.

5) Identical values of the costs obtained using the same filter indicate similarity of the RMSE performances. From the RMSE plots for KF (Fig.2) and EKF (Fig.3), it is observed that similar robust performances are obtained for $n_q > 4.21$ while similar sensitive performances are obtained for $n_q < (-4.79)$. For the KF filter, the position estimates show lower RMSE but with a tendency of divergence during the end phase whereas the velocity estimates have additional initial spikes in the sensitive zone of n_q while the RMSE performances in this zone are much improved in the case of the EKF. However, it is interesting to note that the performances of both the KF and the EKF filters are quite similar in the robust zone. It is also observed that at $n_q = 7.21$, the cost J_2 for both the filters shows a high spike, with values of 1.75 and 1.76 for KF and EKF respectively. From the RMSE plots, it is observed that this corresponds to high initial peaks in the position estimates in both filters.

6) In the zone where the values of J_1 and J_2 are changing and specifically near the crossover point, the trade-off between sensitiveness and robustness of the filter can be achieved with a judicious choice of the combination of the tuning parameters. A non-judicious choice may lead to filter instability. So, the best compromise in the RMSE performances is expected near $Q_0 = Q_{comp}$. This is validated from the RMSE plots for both filters for $0.21 \leq n_q \leq 1.21$, where there are no sharp peaks in the initial phase nor is there any divergent behaviour at the end phase.

V. Conclusion

In the present work, two new costs J_1 and J_2 have been proposed for determining the filter tuning parameters on the basis of the innovation covariance S_k by using the concept of the estimated *measurement* and the factors contributing to it. It is to be noted that the standard treatment in the existing literature [12], [15], [16] focuses simply on the estimated states and the errors thereof. So, the proposed approach provides a major shift in the filtering paradigm.

For predicting the proper combination of the filter tuning parameters, these performance indices have been evaluated for a 2D ballistic target problem [17] as shown in Fig.1 and Table I. For this, the critical tuning parameter Q [1], and hence, a new controlling parameter n_q, as defined in eqn. (13), has been varied by considering $Q_0 = 10^p Q_{nom}$ where Q_{nom} is a nominal Q matrix and the multiplier p is varied suitably. All the other filter tuning parameters, x_0, P_0^+ and R, are kept fixed. The predictions from the costs J_1 and J_2, have been validated in terms of the RMSE performances for KF (Fig.2) and EKF (Fig.3) and are summarized hereafter.

It is observed that both J_1 and J_2 are bounded by the number of measurements $m = 2$ in the upper limit and 0 in the lower limit for significantly high and low values of n_q. As n_q changes, the values of both J_1 and J_2 change between these limits of m and 0. While P_0 and R are fixed, J_1 attains a steady maximum value close to m for low n_q while J_2 reaches a maximum value close to m for high n_q. A minimum value, very close to 0, is obtained for J_2 when n_q is very small and for J_1, when n_q is very large, for both KF and EKF.

At a particular value of n_q, with the corresponding value of Q_0 henceforth referred to as the compromise value, Q_{comp}, there is a crossover of the plots for J_1 and J_2. In this case, the corresponding value of n_q for Q_{comp} lies between 0.21 and 1.21 for both KF and EKF. A value of J_2 close to the number of measurements m indicates that the filter exhibits robustness for those combinations of the tuning parameters. On the other hand, when the value of J_1 is close to the number of measurements, this indicates a sensitivity in the RMSE performance but this might also cause the filter to diverge. Hence, robustness of the RMSE performances are expected and obtained for KF and EKF for $n_q \geq 1.21$ while sensitivity of performances of the filters increase for $n_q < 1$. However, there is a limit to both of these since identical costs for the same filter yields identical RMSE performances. So, the proposed costs J_1 and J_2 can be used by the design engineer to decide suitable choices of the filter tuning parameters based on the performance requirements for the system.

Further studies have been and are being performed by the present researchers using these predictive costs for different linear and nonlinear system and measurement scenarios which validate the observations stated in this work. These also provide additional insight into the choice of the filter tuning parameters and their effects on the RMSE performances.

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References

[1] M. Ananthasayanam, "Kalman Filter Design by Tuning its Statistics or Gains?" in *International Conference on Data Assimilation*, IISc, Bangalore, July 2011.

[2] J. Cordova Alarcon, H. Rodriguez Cortes, and E. Vivas, "Extended Kalman Filter Tuning in Attitude Estimation from Inertial and Magnetic Field Measurements," in *Electrical Engineering, Computing Science and Automatic Control,CCE,2009 6th International Conference on*, Jan. 2009, pp. pp 1–6.

[3] M. Ananthasayanam, A. Sarkar, and S. Vathsal, "Improved adaptive estimation of noise covariance in design of trajectory tracking filters," in *AIAA Atmospheric Flight Mechanics Conference and Exhibit*, Monterey, Caliornia, August 2002, pp. 5–8.

[4] T. Powell, "Automated tuning of an Extended Kalman Filter using the downhill simplex algorithm," *Journal of Guidance, Control and Dynamics*, vol. 25, no. 5, pp. 901–908, 2002.

[5] O. V. Korniyenko, M. S. Sharawi, and D. N. Aloi, "Neural Network Based Approach for Tuning Kalman Filter," in *IEEE International Conference on Electro Information Technology*, vol. 25, no. 5, May 2005, pp. pp 1–5.

[6] R. Clements, P. Tavares, and P. Lima, "Small Satellite Attitude Control Based on a Kalman Filter," in *Proceedings of the 2000 IEEE International Symposium on Intelligent Control*, 2000, pp. 79–84.

[7] R. Rahbari, B. W. Leach, J. Dillon, and C. W. de Silva, "Adaptive tuning of a Kalman filter using the fuzzy integral for an intelligent navigation system," in *Proceedings of IEEE International Intelligent Control Symposium*, 2002, pp. 252–257.

[8] J. A. Rosendo, A. Bachiller, and A. Gomez, "Application of Self-Tuned Kalman Filters to Control of Active Power Filters," in *Proceedings of IEEE Power Tech*, Lausanne, July 2007, pp. pp 1262–1265.

[9] S. Bolognani, L. Tubiana, and M. Zigliotto, "Extended Kalman Filter Tuning in Sensorless PMSM Drives," *IEEE Transactions on Industry Applications*, vol. 39, no. 6, pp. 1741–1747, 2003.

[10] B. M. Åkesson, J. B. Jørgensen, N. K. Poulsen, and S. B. Jørgensen, "A Tool for Kalman Filter Tuning," in *17th European Symposium on Computer Aided Process Engineering - ESCAPE17*, 2007.

[11] T. Kailath, "An innovations approach to least squares estimation-Part I: Linear Filtering in additive white noise," *IEEE Transactions on Automatic Control*, vol. AC-13, no. 6, pp. pp 646–655, December 1968.

[12] Y. Bar Shalom, X. Li, and T. Kirubarajan, *Estimation with applications to Tracking and Navigation*. John Wiley and Sons, New York, 2001.

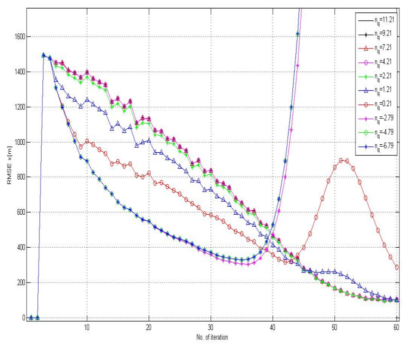
[13] A. Paramonova, M. Kleinhof, and Y. Paramonov, "Normal, Lognormal, Weibull OR P-Sev distribution of Ultimate Strngth of Strands and Composite Specimens," in *Proceedings of the 5th International Conference RelStat05 Part I*, vol. 7, no. 1, 2006, pp. pp 50–59.

[14] P. Zarchan and H. Musoff, *Fundamentals of Kalman Filtering: A Practical Approach*. American Institute of Aeronautics and Astronautics, Virginia, 2000.

[15] R. Brown and P. Hwang, *Introduction to Random Signals and Applied Kalman Filtering*. John Wiley and Sons, New York, 1996.

[16] D. Simon, *Optimal State Estimation: Kalman, H_∞ and Nonlinear Approaches*. Wiley Interscience, 2006.

[17] A. Farina, B. Ristic, and D. Benvenuti, "Tracking a ballistic target: Comparison of several filters," *IEEE Transactions on Aerospace and Electronics System*, vol. 38, 2002.



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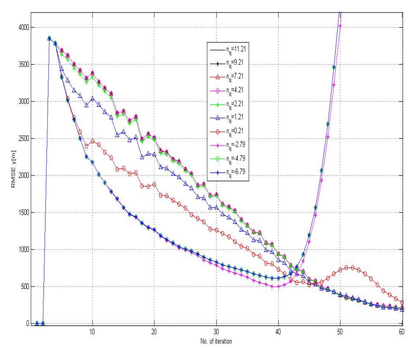

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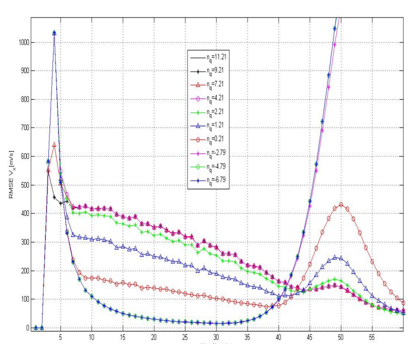

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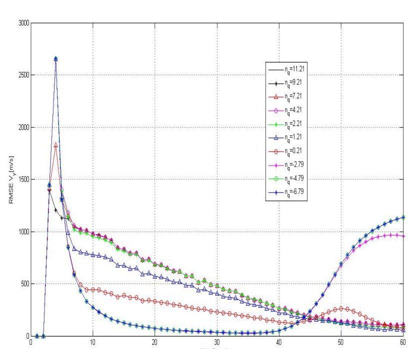

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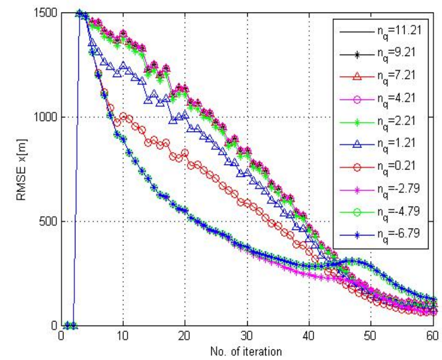
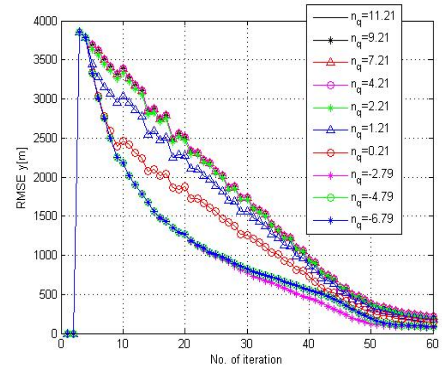
Fig. 2. RMSE plots of the positions a)x and b)y and velocities c)V_x and d)V_y using KF



(a)



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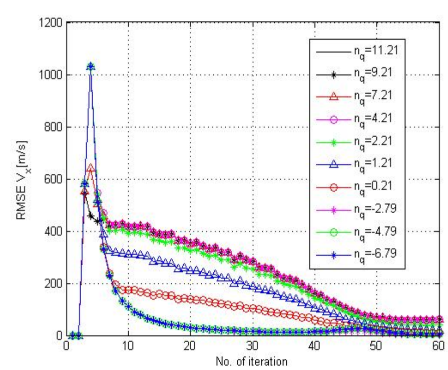
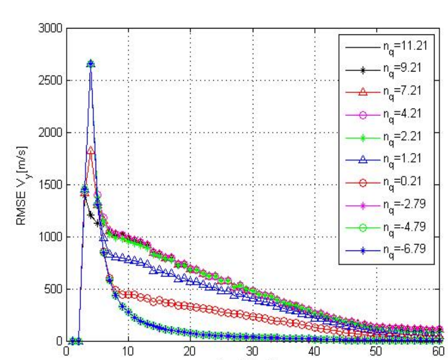


(c)



(d)

Fig. 3. RMSE plots of the positions a)x and b)y and velocities c)V_x and d)V_y using EKF